Exercise 12

For the following exercises, sketch the parametric equations by eliminating the parameter. Indicate any asymptotes of the graph.

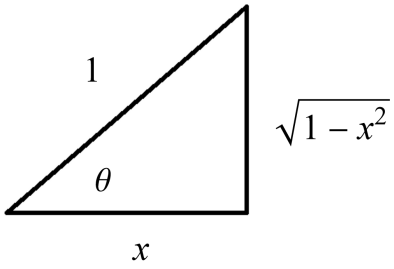
$$x = \cos\theta, \quad y = 2\sin(2\theta)$$

Solution

The equation for x can be written as

$$\frac{x}{1} = \cos\theta.$$

Draw the implied right triangle and use the Pythagorean theorem to determine the missing side.



As a result,

$$\begin{aligned} y &= 2\sin(2\theta) \\ &= 2(2\sin\theta\cos\theta) \\ &= 4\sin\theta\cos\theta \\ &= 4\left(\frac{\sqrt{1-x^2}}{1}\right)\left(\frac{x}{1}\right) \\ &= 4x\sqrt{1-x^2}. \end{aligned}$$

Square both sides.

$$y^2 = 16x^2(1-x^2)$$

Below is a plot of the parametric equations for $0 \leq t \leq 2\pi$.